

# On the non-perturbative corrections for $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ in Heavy Quark Effective Theory<sup>\*</sup>

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**Abstract** We study consequences of the non-forward amplitude for the baryon decay  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$  which will be measured in detail at *LHCb*. We obtain a sum rule for the subleading elastic Isgur-Wise (IW) function  $A(w)$  that originates from the kinetic part of the  $O(1/m_Q)$  effective Lagrangian perturbation. In the sum rule appear only the intermediate states  $J_j^P = \frac{1}{2}_0^+$ , the same that contribute to the  $O(1/m_Q^2)$  correction to the axial-vector form factor  $G_1(w)$  involved in the differential decay rate at zero recoil  $w=1$ . This allows us to obtain a lower bound on the correction  $-\delta_{1/m_Q^2}^{(G_1)}$  in terms of  $A(w)$  and the shape of the leading elastic IW function  $\xi_\Lambda(w)$ . Another theoretical implication is that  $A'(1)$  must vanish in the limit where the slope  $\rho_\Lambda$  of  $\xi_\Lambda(w)$  saturates its lower bound. A strong correlation between the leading IW function  $\xi_\Lambda(w)$  and the subleading one  $A(w)$  is thus established in the case of the baryons.

**Key words** HQET, Isgur-Wise functions, baryon spectroscopy

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## 1 Introduction

We aim at investigating within the Heavy Quark Effective Theory formalism (HQET) the  $O(1/m_Q^2)$  subleading corrections to the baryonic semileptonic decay  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ <sup>[1]</sup>. An important ingredient is the consideration of the non-forward amplitude  $\Lambda_b(v_i) \rightarrow \Lambda_c(v') \rightarrow \Lambda_b(v_f)$  allowing for general velocities  $v_i$ ,  $v_f$  and  $v'$ . At leading order, the HQET sum rule method has been applied to the case of the baryonic leading elastic IW function  $\xi_\Lambda(w) = 1 - \rho_\Lambda^2(w-1) + \frac{\sigma_\Lambda^2}{2}(w-1)^2 + \dots$ , giving the following lower bounds for its slope<sup>[2]</sup> and its curvature<sup>[3]</sup>:

$$\rho_\Lambda^2 = \sum_{n \geq 0} [\tau_1^{(n)}(1)]^2 \geq 0, \quad (1)$$

$$\sigma_\Lambda^2 = \frac{3}{5} \left( \rho_\Lambda^2 + (\rho_\Lambda^2)^2 + \sum_{n \neq 0} [\xi_\Lambda^{(n)'}(1)]^2 \right) \geq \frac{3}{5} (\rho_\Lambda^2 + (\rho_\Lambda^2)^2) \quad (2)$$

where  $\tau_1^{(n)}(1)$  denote the IW functions of transition  $j^P = 0^+ \rightarrow 1^-$  at zero recoil ( $j^P$  is the spin-parity of the so-called *brown muck*, namely, all the light degrees of freedom within the heavy-light baryon, and  $n$  is a radial quantum number).

This leading as well as the new subleading results

presented in this letter consist of a set of constraints that the differential decay rate of  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$  - which will be accurately measured at *LHCb* - must fulfill. Data has already been obtained for this mode at the Tevatron giving a large branching ratio of about 5% and a large fraction of the inclusive semileptonic decay  $BR(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell + \text{anything}) \simeq 10\%$ .

The proceeding is organized as follows. We first introduce the leading and subleading IW functions  $\xi_\Lambda(w)$  and  $A(w)$  relevant for the  $O(1/m_Q^2)$  non-perturbative correction  $\delta_{1/m_Q^2}^{(G_1)}$  to the axial-vector form factor  $G_1(1)$  at zero recoil which enters into the expression of the differential rate of the semileptonic decay  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ . In Sections 3 and 4, we derive sum rules respectively for the subleading elastic IW function  $A(w)$  and for the correction  $\delta_{1/m_Q^2}^{(G_1)}$ . Section 5 presents a bound for the latter and, in Section 6, we discuss the strong correlation between leading and subleading baryonic quantities in HQET.

## 2 Leading and subleading baryonic IW functions

Of particular interest for us will be the form factor  $G_1(w)$  which enters into the parametrization of the

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baryonic matrix element of the axial-vector current<sup>[4]</sup>:

$$\begin{aligned} \langle \Lambda_c(v', s') | \bar{c} \gamma^\mu \gamma_5 b | \Lambda_b(v, s) \rangle = \\ \bar{u}_{\Lambda_c}(v', s') \left[ G_1(w) \gamma^\mu + G_2(w) v^\mu + G_3(w) v'^\mu \right] \gamma_5 u_{\Lambda_b}(v, s) \end{aligned} \quad (3)$$

where  $w \equiv v \cdot v'$  is the recoil and  $u_{\Lambda_Q}(v, s)$  is the spinor of the heavy baryon physical state such that  $\not{v} u_{\Lambda_Q}(v, s) = u_{\Lambda_Q}(v, s)$  and  $\bar{u}_{\Lambda_Q}(v, s) u_{\Lambda_Q}(v, r) = 2m_{\Lambda_Q} \delta_{rs}$  ( $p_{\Lambda_Q}^\mu = m_{\Lambda_Q} v^\mu$  with  $m_{\Lambda_Q}$  the physical mass of the heavy-light baryon  $\Lambda_Q$ ).

Indeed, in the neighborhood of the zero recoil  $w = 1$  kinematic point, the differential rate of the transition  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$  depends only on  $G_1(1)$ :

$$\frac{1}{\sqrt{w^2 - 1}} \frac{d\Gamma}{dw} \underset{w \simeq 1}{\simeq} \frac{G_F^2 |V_{cb}|^2}{4\pi^3} m_{\Lambda_c}^3 (m_{\Lambda_b} - m_{\Lambda_c})^2 |G_1(1)|^2. \quad (4)$$

In the heavy quark expansion, the form factor  $G_1(w)$  is expressed, at the order  $O(1/m_Q)$  ( $Q = b$  or  $c$  quark), in terms of two IW functions  $\xi_\Lambda(w)$  and  $A(w)$ :

$$G_1(w) = \xi_\Lambda(w) + \left( \frac{1}{2m_b} + \frac{1}{2m_c} \right) \left[ \frac{w-1}{w+1} \bar{\Lambda} \xi_\Lambda(w) + A(w) \right]. \quad (5)$$

The leading elastic IW function  $\xi_\Lambda(w)$  is defined by the matrix element of the lowest-order heavy-heavy current  $J = \bar{h}_{v'}^{(Q')} \Gamma h_v^{(Q)}$  in HQET ( $\Gamma$  is any combination of gamma matrices):

$$\langle \Lambda_c(v', s') | \bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} | \Lambda_b(v, s) \rangle = \xi_\Lambda(w) \bar{U}_{\Lambda_c}(v', s') \Gamma U_{\Lambda_b}(v, s) \quad (6)$$

where  $U_{\Lambda_Q}(v, s) = (1 + O(1/m_Q^2))^{-1/2} u_{\Lambda_Q}(v, s)$  is the spinor of the effective heavy baryon state in HQET normalized such that  $\bar{U}_{\Lambda_Q}(v, s) U_{\Lambda_Q}(v, r) = 2M_{\Lambda_Q} \delta_{sr}$ . The effective mass of the HQET state  $M_{\Lambda_Q} = m_Q + \bar{\Lambda}$  is given in terms of the energy  $\bar{\Lambda}$  of the *brown muck*.  $h_v^{(Q)}(x) = e^{im_Q v \cdot x} \left( \frac{1+\not{v}}{2} \right) Q(x)$  is the effective heavy quark field which appears when building the effective Lagrangian as a power series expansion in  $1/m_Q$ :

$$\mathcal{L}_{eff}^{(Q)} = \mathcal{L}_{HQET,v}^{(Q)} + \mathcal{L}_{kin,v}^{(Q)} + \mathcal{L}_{mag,v}^{(Q)} \quad (7)$$

with\*

$$\begin{cases} \mathcal{L}_{HQET}^{(Q)} = \bar{h}_v^{(Q)} (i v \cdot D) h_v^{(Q)}, \\ \mathcal{L}_{kin,v}^{(Q)} = \frac{1}{2m_Q} \mathcal{O}_{kin,v}^{(Q)} ; \mathcal{O}_{kin,v}^{(Q)} = \bar{h}_v^{(Q)} (i D_\perp)^2 h_v^{(Q)}, \\ \mathcal{L}_{mag}^{(Q)} = \frac{1}{2m_Q} \mathcal{O}_{mag,v}^{(Q)} ; \mathcal{O}_{mag,v}^{(Q)} = -\frac{g_s}{2} \bar{h}_v^{(Q)} \sigma_{\mu\nu} G^{\mu\nu} h_v^{(Q)}. \end{cases} \quad (8)$$

As for  $A(w)$ , it stems from the kinetic part of the  $O(1/m_Q)$  effective Lagrangian:

$$\langle \Lambda_c(v', s') | i \int d^4x T \{ J(0), \mathcal{O}_{kin}^{(Q)}(x) \} | \Lambda_b(v, s) \rangle = A(w) \bar{U}_{\Lambda_c} \Gamma U_{\Lambda_b}. \quad (9)$$

It is worth pointing out that, contrary to the case of the meson ground-state doublet  $j^P = \frac{1}{2}^-$  where the transition matrix element of  $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$  gets two *Current*-type and three *Lagrangian*-type  $O(1/m_Q)$  corrections (the latter coming both from  $\mathcal{L}_{kin}$  and  $\mathcal{L}_{mag}$ )<sup>[5-7]</sup>, in the case of the baryon ground-state singlet  $j^P = 0^+$  considered here,  $G_1$  gets only one *Current*-type corrections (in terms of  $\bar{\Lambda} \xi_\Lambda$ ) and only one *Lagrangian*-type correction (in terms of  $A(w)$ ) as the magnetic part  $\mathcal{O}_{mag,v}^{(Q)}$  of the  $O(1/m_Q)$  Lagrangian perturbation doesn't contribute to  $A(w)$ <sup>[8]</sup>.

Furthermore, due to the vector current conservation, we have the condition  $A(1) = 0$  at zero recoil<sup>[4]</sup>. Consequently, according to (5), the form factor  $G_1(1)$  in (4) has, at zero recoil, only  $O(1/m_Q^2)$  corrections which we will denote as follows:

$$G_1(1) = 1 + \delta_{1/m_Q^2}^{(G_1)}. \quad (10)$$

The main goal of the present letter is to study these corrections<sup>[1]</sup>.

### 3 HQET sum rule for the subleading IW function $A(w)$

Since only the kinetic perturbation  $\mathcal{O}_{kin}^{(Q)}$  of the effective Lagrangian contributes to the definition (9) of  $A(w)$ , only the radially excited intermediate states  $\Lambda_c^{(n)}$  with the same spin-parity  $J_j^P = \frac{1}{2}_0^+$  as the ground-state  $\Lambda_c$  are relevant and give us the following sum rule:

$$A(w) = \sum_{n \neq 0} \frac{\xi_\Lambda^{(n)}(w)}{\Delta E^{(n)}} \frac{\langle \Lambda_c^{(n)}(v, s) | \mathcal{O}_{kin,v}^{(c)}(0) | \Lambda_c(v, s) \rangle}{\sqrt{4m_{\Lambda_c^{(n)}} m_{\Lambda_c}} \sqrt{v_{\Lambda_c^{(n)}}^0 v_{\Lambda_c}^0}} \quad (11)$$

where  $\Delta E^{(n)} \equiv m_{\Lambda_c^{(n)}} - m_{\Lambda_c}$ . Especially, we recover  $A(1) = 0$  since  $\xi_\Lambda^{(n)}(1) = \delta_{n,0}$ .

### 4 HQET sum rule for the correction $\delta_{1/m_Q^2}^{(G_1)}$ to the form factor $G_1(1)$

Analogously to the sum rule formulated in the mesonic case for the axial-vector current (Eqns (114) and (5.6) of <sup>[9]</sup> and <sup>[10]</sup> respectively), we have for the form factor  $G_1$  at the order  $O(1/m_Q^2)$ :

\*The perpendicular component of the covariant derivative  $D$  is defined by  $D_{\mu\perp} = D_\mu - (v \cdot D) v_\mu$ . Also,  $[D_\mu, D_\nu] = i g_s G_{\mu\nu}$  and  $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ .

$$|G_1(1)|^2 + \frac{1}{2} \sum_{s,s'} \sum_{n \neq 0} \frac{|\langle \Lambda_c^{(n)}(0^+, 1^+)(v, s') | \vec{A} | \Lambda_b(0^+)(v, s) \rangle|^2}{4m_{\Lambda_c^{(n)}} m_{\Lambda_b}} = 1 + \left[ \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \frac{8}{3} \frac{1}{2m_c} \frac{1}{2m_b} \right] \lambda \quad (12)$$

where the hard radiative corrections have been neglected and where  $-\lambda = \frac{1}{2M_{\Lambda_b}} \langle \Lambda_b(v) | \bar{h}_v^{(b)} (iD_\perp)^2 h_v^{(b)} | \Lambda_b(v) \rangle$  is the mean kinetic energy value. The  $O(1/m_Q^2)$  correction to  $G_1$  takes then the following expression:

$$-\delta_{1/m_Q^2}^{(G_1)} = -\frac{1}{2} \left[ \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \frac{8}{3} \frac{1}{2m_c} \frac{1}{2m_b} \right] \lambda + \frac{1}{4} \sum_{s,s'} \sum_{n \neq 0} \frac{|\langle \Lambda_c^{(n)}(0^+)(v, s') | \vec{A} | \Lambda_b(v, s) \rangle|^2}{4m_{\Lambda_c^{(n)}} m_{\Lambda_b}} + \frac{1}{4} \sum_{s,s'} \sum_{n \neq 0} \frac{|\langle \Lambda_c^{(n)}(1^+)(v, s') | \vec{A} | \Lambda_b(v) \rangle|^2}{4m_{\Lambda_c^{(n)}} m_{\Lambda_b}}. \quad (13)$$

The matrix elements implicitly contain the double insertion (on the initial  $b$ - and final  $c$ -legs) of the kinetic and magnetic parts of the  $O(1/m_Q)$  effective Lagrangian to the HQET axial-vector current  $A^\mu \equiv \bar{h}_v^{(c)} \gamma^\mu \gamma_5 h_v^{(b)}$  for which only the spatial component  $\vec{A}$

survives in the heavy quark limit  $m_Q \rightarrow \infty$ . In (13), the final states  $\Lambda_c^{(n)}(J_j^P = \frac{1}{2}_0^+)$  and  $\Lambda_c^{(n)}(J_j^P = \frac{1}{2}_1^+, \frac{3}{2}_1^+)$  are attained respectively by  $\mathcal{L}_{kin}$  and  $\mathcal{L}_{mag}$ . For the sake of argument, we have:

$$\langle \Lambda_c^{(n)}(v, s') | \bar{h}_v^{(c)} \vec{A} h_v^{(b)}(0) | \Lambda_b(v, s) \rangle = \frac{1}{2m_c} \langle \Lambda_c^{(n)}(v, s') | i \int d^4 x T \{ \mathcal{O}_{kin,v}^{(c)}(x), \bar{h}_v^{(c)} \vec{A} h_v^{(b)}(0) \} | \Lambda_b(v, s) \rangle \quad (14)$$

$$+ \frac{1}{2m_b} \langle \Lambda_c^{(n)}(v, s') | i \int d^4 x T \{ \mathcal{O}_{kin,v}^{(b)}(x), \bar{h}_v^{(c)} \vec{A} h_v^{(b)}(0) \} | \Lambda_b(v, s) \rangle \quad (15)$$

which, by inserting the intermediate states and using the flavor-spin heavy quark symmetry, finally gives:

$$\langle \Lambda_c^{(n)}(v, s') | \bar{h}_v^{(c)} \vec{A} h_v^{(b)}(0) | \Lambda_b(v, s) \rangle \stackrel{O(1/m_Q)}{=} \frac{-1}{\Delta E^{(n)}} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) \frac{\langle \Lambda_c^{(n)}(v, s) | \mathcal{O}_{kin,v}^{(c)}(0) | \Lambda_c(v, s) \rangle}{\sqrt{4m_{\Lambda_c^{(n)}} m_{\Lambda_c}} \sqrt{v_{\Lambda_c^{(n)}}^0 v_{\Lambda_c}^0}} \mathcal{U}_{\Lambda_c}^\dagger(v, s') \vec{\Sigma} \mathcal{U}_{\Lambda_b}(v, s) \quad (16)$$

where  $\vec{\Sigma} = \text{diag}(\vec{\sigma}, \vec{\sigma})$  is the double Pauli matrix.

A similar evaluation of the second matrix element in (13) between the ground-state singlet  $\Lambda_b(\frac{1}{2}_0^+)$  and the excited states  $\Lambda_c^{(n)}(\frac{1}{2}_1^+, \frac{3}{2}_1^+)$  gives a positive semi-definite contribution and, in particular, a term proportional to  $1/(4m_c m_b)$  reminiscent of the one found in<sup>[4]</sup>:

$$G_1(1) = 1 + \frac{1}{2} \left[ \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \frac{8}{3} \frac{1}{2m_c} \frac{1}{2m_b} \right] \lambda + \frac{1}{2} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 [-D_1(1) + 3D_2(1)] + \frac{1}{2m_c} \frac{1}{2m_b} 4D_2(1) \quad (17)$$

where the functions  $D_i$  ( $i=1,2$ ) correspond to the double insertion of the operators  $\mathcal{O}_{kin,v}^{(Q)}$  and  $\mathcal{O}_{mag,v}^{(Q)}$  on the initial and final heavy quark legs ( $Q=b,c$ ).

## 5 Bound on the corrections $\delta_{1/m_Q^2}^{(G_1)}$

The preceding analysis allows us to write a bound for the  $O(1/m_Q^2)$  corrections to  $G_1(1)$ . Eqn. (13) implies indeed that

$$-\delta_{1/m_Q^2}^{(G_1)} \geq -\frac{1}{2} \left[ \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \frac{8}{3} \frac{1}{2m_c} \frac{1}{2m_b} \right] \lambda + \frac{1}{2} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \sum_{n \neq 0} \left[ \frac{1}{\Delta E^{(n)}} \frac{\langle \Lambda_c^{(n)}(v, s) | \mathcal{O}_{kin,v}^{(c)}(0) | \Lambda_c(v, s) \rangle}{\sqrt{4m_{\Lambda_c^{(n)}} m_{\Lambda_c}} \sqrt{v_{\Lambda_c^{(n)}}^0 v_{\Lambda_c}^0}} \right]^2. \quad (18)$$

The crucial point here is that the intermediate states entering into (18) are the same that the ones contributing to the sum rule (11) for  $A(w)$ . Using the Cauchy-Schwarz inequality  $|\sum_n A_n B_n|^2 \leq (\sum_n |A_n|^2)(\sum_n |B_n|^2)$ , the latter gives:

$$[A(w)]^2 \leq \sum_{n \neq 0} [\xi_\Lambda^{(n)}(w)]^2 \sum_{n \neq 0} \left[ \frac{1}{\Delta E^{(n)}} \frac{\langle \Lambda_c^{(n)}(v, s) | \mathcal{O}_{kin, v}^{(c)}(0) | \Lambda_c(v, s) \rangle}{\sqrt{4m_{\Lambda_c^{(n)}} m_{\Lambda_c}} \sqrt{v_{\Lambda_c^{(n)}}^0 v_{\Lambda_c}^0}} \right]^2 \quad (19)$$

such that

$$-\delta_{1/m_Q^2}^{(G_1)} \geq -\frac{1}{2} \left[ \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \frac{8}{3} \frac{1}{2m_c} \frac{1}{2m_b} \right] \lambda + \frac{1}{2} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \frac{[A(w)]^2}{\sum_{n \neq 0} [\xi_\Lambda^{(n)}(w)]^2}. \quad (20)$$

Since this inequality is valid for any value of  $w$ , we can consider its limit  $w \rightarrow 1$ , taking into account that  $A(1) = 0$  and  $\xi_\Lambda^{(n)}(1) = \delta_{n,0}$ :

$$-\delta_{1/m_Q^2}^{(G_1)} \geq -\frac{1}{2} \left[ \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \frac{8}{3} \frac{1}{2m_c} \frac{1}{2m_b} \right] \lambda + \frac{1}{2} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \frac{[A'(1)]^2}{\sum_{n \neq 0} [\xi_\Lambda^{(n)'}(1)]^2} \quad (21)$$

and from (2),  $\sum_{n \neq 0} [\xi_\Lambda^{(n)'}(1)]^2 = \frac{5}{3} \sigma_\Lambda^2 - \rho_\Lambda^2 - (\rho_\Lambda^2)^2$ , one finally gets:

$$-\delta_{1/m_Q^2}^{(G_1)} \geq -\frac{1}{2} \left[ \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \frac{8}{3} \frac{1}{2m_c} \frac{1}{2m_b} \right] \lambda + \frac{3}{10} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \frac{[A'(1)]^2}{\sigma_\Lambda^2 - \frac{3}{5}[\rho_\Lambda^2 + (\rho_\Lambda^2)^2]} \quad (22)$$

which is the main result of the proceeding. From Eqns (4) and (10), we see that the  $O(1/m_Q^2)$  correction at zero recoil  $-\delta_{1/m_Q^2}^{(G_1)}$  is pivotal in the extrapolation of the semileptonic differential decay rate  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$  near zero recoil. In particular, this is needed to check that the value of  $|V_{cb}|$  that would fit exclusive baryon semileptonic data is indeed consistent with what we presently know on this parameter from the meson exclusive and inclusive determinations. It is in this respect that the bound (22) is important.

## 6 Correlation between $A'(1)$ and the shape of the leading elastic IW function $\xi_\Lambda(w)$

Since the inequality (22) holds for any values of  $\rho_\Lambda^2$  and  $\sigma_\Lambda^2$  satisfying the constraints (1) and (2), it should hold for their lowest values. However, if the curvature attains its lowest value  $\sigma_\Lambda^2 \rightarrow \frac{3}{5}(\rho_\Lambda^2 + (\rho_\Lambda^2)^2)$ , the second term on the *r.h.s.* of (22) would diverge. Because this behavior is unphysical, we predict instead a strong correlation between  $A'(1)$  and the shape of the

leading elastic IW function  $\xi_\Lambda(w)$ . Eqn. (22) implies the correlation:

$$\text{if } \sigma_\Lambda^2 \Rightarrow \frac{3}{5}(\rho_\Lambda^2 + (\rho_\Lambda^2)^2) \text{ then } A'(1) \Rightarrow 0. \quad (23)$$

As a matter of fact, a group theory-based method (equivalent to the HQET sum rule approach) has been developed to study the baryonic IW functions<sup>[11]</sup>. It turns out that if the slope of  $\xi_\Lambda(w)$  attains its lowest possible value  $\rho_\Lambda^2 = 0$ , then all the higher-order derivatives  $\xi_\Lambda^{(n)}(1)$  ( $n \geq 2$ ) vanish at zero recoil. Especially, that implies  $\sigma_\Lambda^2 = 0$ . In view of (23), we have then:

$$\text{if } \rho_\Lambda^2 \Rightarrow 0 \text{ then } A'(1) \Rightarrow 0. \quad (24)$$

The non-trivial results (23) and (24) relate the behavior of the leading elastic IW function  $\xi_\Lambda(w)$  to the subleading effective form factor  $A(w)$ .

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